

Blackholes

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Recommended book: *Gravity From the Ground Up* by Schutz (ISBN: 0 521 45506 5). How gravity works, starting with Newton and working forward
Book from the lecture: *Gravitation* by Misner (sp?) (available online)

1 Laws of Gravity

1.1 Newton

$$m_i \frac{d^2 \vec{x}}{dt^2} = \vec{F} = m_i \vec{g} \quad (1)$$

$$\vec{g} = \sum_j \frac{-G m_j (\vec{x} - \vec{x}_j)}{|\vec{x} - \vec{x}_j|^3} \quad (2)$$

$$= \nabla \phi(\vec{x}_j) \quad (3)$$

$$\phi(\vec{x}_j) = -G \int \frac{\rho(\vec{x}_j) d^3 x}{|\vec{x}_j - \vec{x}|} \quad (4)$$

$$\nabla^2 \phi = 4\pi G \rho(\vec{x}_j) \quad (5)$$

1.2 Einstein

$$r = \frac{GM}{3c^2} \quad (6)$$

$$\text{Let } M = \frac{4}{3} \pi r^2 \rho \quad (\text{where } \rho \text{ is uniform density (average over } r)) \quad (7)$$

$$f_r = f_e \left(1 + \frac{gH}{c^2}\right) \quad (8)$$

$$(9)$$

2 Dark Stars

Rev John Michell (1783) suggests that a giant star could keep even light from escaping.

Escape velocity

$$\frac{1}{2}mv^2 = \frac{GMm}{r} \tag{10}$$

$$v = \sqrt{\frac{2GM}{r}} \tag{11}$$

3 Einstein's Ideas

Equivalence principle: Can't distinguish a smoothly accelerating elevator (or rocket) from gravity pulling on you.

Gravity as curvature: Gravity curves light path and time (slows it down):
 $G_{uv} + \Lambda g_{uv} = 8\pi T_{uv}$ (G = geometry of space time, Λ = cosmological constant, T = distribution of matter, motion in spacetime)

3.1 Spacetime

Constant time = hyperbola

$$(ct_B)^2 = (ct_A)^2 + X_A^2 \tag{12}$$

$$\tag{13}$$

Mathematical tool that handles space and time $P(t, x, y, z)$.

Ex: 2D space:

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 \quad \text{Euclidean Cartesian space} \tag{14}$$

$$(\Delta s)^2 = -(\Delta t)^2 + (\Delta x)^2 \quad \text{Minkowski Cartesian space} \tag{15}$$

$$\tag{16}$$

A body will move along the curve of shortest distance (geodesic) in *spacetime* unless a force acts on it (not generally shortest distance in normal space).

In general, geodesics are difficult to calculate.

Easy if you embed in Euclidean space.

Metric: converts flat map distances into actual distances. Denoted by "g".

Finding a useful metric is hard — then you just slot it into a standard equation.

Non-physics example: Taxi metric to convert time, location, etc. into a cab ride cost.

Special relativity: Matter moves at most as fast as light. Stuck inside a cone limiting speed.

3.2 Metric g in four dimensions

In four dimensions, need 10 numbers to indicate curvature.

Need additional math tools to keep them straight.

Call it a *tensor*. Organize groups of tensors.

- 0-tensor: single number, eg 5
- 1 tensor: 4 numbers (vector), eg $A_j = (1, 0, -\pi, 2)$
- 2-tensor: $4 \times 4 = 16$ number (matrix), eg... eh, you know...

$$G_{uv} + \Lambda g_{uv} = 8\pi T_{uv} \quad (17)$$

$$G_{uv} = R_{ij} - \frac{1}{2}g_{uv}R \quad (\text{Einstein's}) \quad (18)$$

$$T_{uv} = \begin{array}{l} \text{stress-energy tensor — describes the density, flux} \\ \text{of energy and momentum in spacetime} \end{array} \quad (19)$$

$$g_{uv} = \text{metric tensor} \quad (20)$$

EFE is a tensor equation relating some 4×4 tensors. Einstein's equations are actually 16 equations of form: $G_{11} = 8\pi G T_{11} + \Lambda g_{11}$

3.3 Verification of GR

In 1919, the sun was seen to have distorted the light from a distant star by about 1.64 arc-seconds. Needed to be done during a solar eclipse because otherwise there would be too much light.